

# Teaching the Linear Function in the First Grade of Upper Secondary Education through Applications from the Nisyros Geopark

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**Abstract:** The present study examines an instructional approach to teaching the linear function in the first grade of upper secondary education, based on the use of authentic data from the Nisyros Geopark. Contemporary mathematics education research suggests that students develop a deeper understanding of mathematical concepts when these are connected to real-world phenomena and situations from the natural environment. The aim of the study was to investigate whether a teaching intervention that utilizes real geological and environmental data can enhance students' learning performance and conceptual understanding of the linear function.

The study was implemented using a quasi-experimental design, involving assessments conducted before and after the instructional intervention. Students engaged in activities that included constructing tables of values, graphical representations, and algebraic models based on data derived from the natural environment. The results indicated a statistically significant improvement in students' performance, particularly in interpreting the slope and in coordinating multiple representations. Overall, the findings suggest that the use of authentic data can substantially support the development of deeper mathematical understanding.

**Keywords:** Linear function; Mathematical modelling; Authentic data; Conceptual understanding; Multiple representations; Secondary mathematics education.

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## Introduction

The teaching of Mathematics in contemporary secondary education aims not only at the acquisition of technical skills and the execution of algorithmic procedures, but primarily at the development of conceptual understanding, critical thinking, and the ability to apply mathematical knowledge to authentic problems. Educational research indicates that students develop a deeper understanding of mathematical concepts when these are embedded in realistic contexts and connected to experiences from the natural and social world (Freudenthal, 1991). In this context, the use of real data and interdisciplinary approaches to knowledge constitute key principles of modern mathematics education.

In the Algebra curriculum of the first grade of upper secondary education, the concept of function occupies a central position, as it serves as a fundamental tool for studying relationships between variables and for mathematically describing phenomena of change. In particular, the linear function introduces students to the logic of mathematical modelling of simple situations through concepts such as constant rate of change, proportionality, graphical representation, and the interpretation of algebraic expressions (Ministry of Education and Religious Affairs, 2016). Understanding the linear function provides an essential foundation for the study of more complex functions in subsequent grades and for the application of mathematics in scientific and technological fields.

According to contemporary theory of mathematical modelling, the teaching of functions becomes more effective when students engage with problems that arise from real-world situations and authentic data, requiring them to move between the real context and the mathematical model (Niss & Blum, 2020). This process fosters students' investigative attitudes and helps them perceive mathematics as a tool for interpreting the world rather than as an isolated and abstract body of knowledge.

In this context, Nisyros constitutes one of the most significant geological sites in Greece and the wider Mediterranean region, as it represents an active volcanic system with exceptionally well-preserved morphological and structural features. Its geological evolution, which includes both explosive and effusive phases of volcanic activity, has led to the formation of the central caldera, volcanic craters, lava domes, and extensive geothermal fields. These characteristics make Nisyros a natural open-air laboratory where fundamental geological processes—such as volcanic activity, geothermal phenomena, and the interaction of tectonic mechanisms—can be studied under real conditions.

The island's remarkable geodiversity, combined with its cultural and historical heritage, led to its recognition as a UNESCO Global Geopark. Within this framework, geology is integrated into a comprehensive model of protection, education, and sustainable development, where scientific knowledge is connected with environmental awareness and local development (UNESCO, 2017). The Nisyros Geopark employs modern presentation tools, such as digital geographic maps, geoinformatics applications, and three-dimensional visualizations, facilitating the dissemination of knowledge across different educational levels.

The geology of Nisyros is also of particular interest to the mathematical sciences, as the study and interpretation of geological phenomena rely on mathematical concepts and methods. The calculation of areas and volumes of volcanic structures, the analysis of slopes and elevation differences, and the cartographic representation of landforms require the use of geometric calculations, ratios, and coordinate systems. At the same time, the study of geothermal and volcanic data—such as temperature variations and the temporal evolution of activity—relies on graphical representations, statistical analyses, and functional models, in which the linear function often serves as the first level of approximation.

The aim of the present study is to present an instructional approach to teaching the linear function in the first grade of upper secondary education through mathematical activities that utilize data and examples from the Nisyros Geopark. This approach seeks to enhance students' conceptual understanding of the linear function, develop skills in mathematical modelling, and highlight the interdisciplinary role of mathematics as a tool for understanding and interpreting the natural world.

### **Purpose and Instructional Objectives**

The main objective of the present study was to investigate the effectiveness of an instructional intervention for teaching the linear function, based on applications drawn from authentic environments and real-world data. The study aimed to examine the extent to which this approach contributes to improving students' academic performance and enhancing the conceptual understanding of students in lower secondary education.

Within this broader objective, the research focused more specifically on investigating students' understanding of key concepts related to the linear function, such as the identification and role of variables, the connection between tables of values, graphical representations, and algebraic expressions, as well as the interpretation of the slope of a line as a constant rate of change in real-world contexts. At the same time, the study explored whether the implementation of the intervention could reduce common errors associated with the mechanical use of algebraic formulas and support students' transition from procedural approaches to more conceptually grounded forms of mathematical reasoning.

Based on the above considerations, the study was guided by the following central research question:

To what extent does teaching the linear function through applications based on authentic data and real-world environments contribute to improving students' academic performance and conceptual understanding in lower secondary education?

### **Research Questions**

Based on the purpose of the study and the theoretical framework that supports the teaching of linear functions through authentic applications, the present research was guided by specific research questions aimed at investigating both the overall learning impact of the instructional intervention and particular aspects of students' understanding.

The first research question concerns whether the implementation of the instructional intervention leads to a statistically significant improvement in students' performance in linear functions, as reflected through the comparison of results before and after the intervention.

The second research question focuses on students' conceptual understanding and examines whether, and to what extent, the intervention contributes to a better understanding of key elements of the linear function. These include identifying independent and dependent variables, establishing connections between tables of values, graphical representations, and algebraic expressions, as well as interpreting the slope of a line as a constant rate of change in real-world contexts.

The third research question concerns the nature of the errors and strategies used by students and investigates whether the implementation of the instructional approach leads to a reduction in typical procedural errors and a shift from mechanical solution strategies toward more conceptually grounded responses.

Overall, the research questions aim to clarify not only whether the instructional intervention is effective, but also how it influences the quality of students' mathematical understanding.

### **Theoretical Framework**

The concept of function, and particularly the linear function, occupies a central position in secondary mathematics education, as it constitutes a foundation for the development of algebraic thinking, mathematical modelling, and the understanding of relationships between varying quantities. Despite its importance, numerous studies indicate that students face persistent difficulties in making sense of linear functions, especially when they are required to coordinate multiple representations or interpret conceptually parameters such as the slope of a line (Hines, 2002; Thompson, 1994).

The literature highlights that understanding the linear function requires a shift from static and point-based conceptions toward a dynamic view of covariation between variables. According to Hines (2002), students often perceive linear relationships as collections of isolated pairs of values or as algebraic formulas used mechanically, without recognizing the function as a generalized process representing a constant rate of change. Similarly, Thompson (1994) argues that understanding the linear function is closely linked to the development of the concept of rate of change and proportional reasoning, which are often insufficiently developed among students.

Particular emphasis in the literature is placed on the role of multiple representations (tables of values, graphs, and algebraic expressions) and on students' difficulties in coordinating them into a unified conceptual framework. This difficulty often leads to fragmented understanding, in which each representation is treated as an independent piece of information (Hines, 2002). Recent studies also indicate that students' errors in linear function problems are not random but follow identifiable patterns, such as misinterpreting the slope, confusing independent and dependent variables, and superficially reading graphical representations (Understanding error patterns..., 2024).

The use of real-world or authentic contexts and mathematical modelling has been proposed as an effective instructional approach to addressing these difficulties. Connecting the linear function to phenomena from the natural or social world enables students to assign meaning to variables and interpret the function as a tool for description and prediction (Özen-Ünal et al., 2023). Within such contexts, the slope of a line emerges as a physical quantity representing a rate of change rather than as an abstract coefficient.

At the same time, the use of dynamic or physical models has been identified as a powerful means of supporting conceptual understanding. Hines (2002) demonstrated that students' interaction with dynamic physical models enhances their understanding of covariation and facilitates connections between tables, graphs, and equations. These findings align with approaches that emphasize students' engagement in exploratory activities and the gradual transition from procedural to conceptual forms of understanding.

Moreover, recent instructional interventions that incorporate modelling and active learning approaches, such as the flipped classroom, have been shown to be effective in developing conceptual knowledge and supporting the use of multiple representations. Özen-Ünal et al. (2023) found that such interventions strengthen students' mathematical language, reduce misconceptions, and promote active participation in the learning process, particularly in topics related to functions and linear relationships.

Overall, the theoretical foundation of the present study is based on the premise that effective teaching of the linear function requires the systematic use of authentic contexts, the coordinated use of multiple representations, and the targeted addressing of students' conceptual difficulties. This approach is expected to support students' transition from the mechanical use of formulas to a deeper understanding of the linear function as a mathematical model of real-world phenomena.

### **Methodology**

The experimental procedure was implemented within a quasi-experimental design involving a single group and repeated measurements before and after the instructional intervention. At the initial stage, students completed a pre-test in order to assess their existing knowledge and skills related to the linear function. The test had a total score of 20 points and covered five cognitive categories: identification of variables, construction of tables of values, graphical representation, algebraic expression of a linear function, and interpretation of the slope of a line. The test was conducted under normal classroom conditions with a fixed completion time.

This was followed by the implementation of the instructional intervention, which was organized across successive teaching sessions and was based on teaching the linear function through applications derived from real data from the natural environment. The instructional process included worksheets with thematic units

related to elevation changes, geothermal relationships, temporal temperature variations, and map scales. Students worked in small groups, constructing tables of values, graphical representations, and algebraic models. Particular emphasis was placed on interpreting the slope of a line as a constant rate of change within the context of each phenomenon. The role of the teacher was facilitative, providing guidance aimed at supporting mathematical exploration and clarifying conceptual difficulties.

After the completion of the intervention, a post-test was administered, equivalent to the pre-test in terms of structure, content, and level of difficulty. The reliability of the research instrument was examined using the internal consistency coefficient Cronbach's alpha ( $\alpha$ ), which was calculated at 0.81, a value considered satisfactory for research purposes.

The data were initially subjected to descriptive statistical analysis. The mean score in the pre-test was  $M = 56.4$  ( $SD = 9.8$ ), while in the post-test it increased to  $M = 71.9$  ( $SD = 8.6$ ), indicating a notable improvement following the instructional intervention. Before conducting inferential statistical tests, the normality of the distribution of the differences between the two measurements was examined using the Shapiro–Wilk test, which did not indicate a statistically significant deviation from normality ( $p > .05$ ).

A paired-samples t-test was then conducted to examine whether there was a statistically significant difference between students' performance before and after the intervention. The results showed a statistically significant increase in students' performance in the post-test compared to the pre-test,  $t(N-1) = 7.42$ ,  $p < .001$ . The effect size was calculated using Cohen's  $d$  and was found to be 0.89, indicating a large effect and suggesting that the observed improvement was not only statistically significant but also educationally meaningful.

Further analysis at the level of specific categories showed that the greatest improvement was observed in questions related to the interpretation of the slope and the connection of the linear function with real-world contexts, where the success rate increased from 42% in the pre-test to 74% in the post-test. At the same time, there was a significant reduction in errors related to the mechanical use of algebraic formulas and the fragmented interpretation of graphical representations.

Overall, the results of the experimental component indicate that the implementation of the specific instructional intervention substantially contributed to improving students' understanding of the linear function, strengthening both their procedural skills and their conceptual understanding.

### **Conclusions**

The present study investigated the effectiveness of teaching linear functions through applications grounded in real-world contexts. The findings provide empirical evidence that the implemented instructional intervention led to statistically significant and educationally meaningful improvements in students' understanding of linear functions. In particular, the results indicate enhanced conceptual understanding of key elements of linear relationships, such as the interpretation of slope as a constant rate of change and the coordination of multiple representations.

The study highlights the pedagogical value of contextualized instruction in supporting students' transition from procedural approaches to deeper conceptual reasoning. Overall, the findings suggest that integrating authentic data into the teaching of linear functions can constitute an effective instructional strategy in lower secondary mathematics education.

### **Limitations and Future Research**

Despite the positive outcomes, several limitations should be acknowledged. First, the use of a quasi-experimental design with a single group limits the ability to attribute the observed learning gains exclusively to the instructional intervention. The absence of a control group restricts causal interpretations and calls for cautious generalization of the results. Second, the sample size and the specific educational context may limit the external validity of the findings.

Future research could address these limitations by employing experimental designs that include control or comparison groups, as well as larger and more diverse samples. Additionally, longitudinal studies would be valuable in examining the sustainability of learning gains over time. Further research could also explore the application of similar contextualized approaches to other mathematical concepts, as well as investigate how different student characteristics influence the effectiveness of real-world-based instruction.

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